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Measurement of high frequency capillary waves on steep gravity waves

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The properties of high frequency capillary waves generated by steep gravity waves on deep water have been measured with a high resolution laser optical slope gauge. The results have been compared with the steady theory of Longuet-Higgins (1963). Good qualitative agreement is obtained. However, the quantitative predictions of the capillary wave slopes cannot be verified by the data because the theory requires knowledge of an idealized quantity – the crest curvature of the gravity wave in the absence of surface tension – which cannot be measured experimentally.

1. Introduction

The phenomenon associated with a deep-water wave as it steepens to a stage near breaking presents a challenging problem in both theoretical and experimental fluid mechanics. The first publication on the subject was by Stokes (1880), who presented a theoretical argument stating that the limiting wave form of a steady, progressive, symmetric gravity wave in the absence of surface tension is one with a corner at the crest, subtending an angle of 120°. Later numerical work by Michell (1893) established that the limiting wave form is attained at a ratio of wave height to wavelength of 0·143. For nearly a century afterwards, theoretical efforts in the subject were confined to attempts to improve the mathematical representation of the near limiting wave.

In a physical situation, the matter is complicated by the presence of surface tension and viscosity. Partly because of this, but mostly because of the highly transient nature of the phenomenon itself, the results in the experimental area were even more limited. In fact, until quite recently (Cox 1958; Miller 1972) there have been no reports of experiments with sufficient accuracy to provide any quantitative information. Cox (1958), in an experimental investigation aimed mainly towards understanding the effects of wind on waves, reported the presence of high frequency capillary waves near the crest of a steep gravity wave even in the absence of wind. This observation apparently motivated Longuet-Higgins (1963) to examine theoretically the effects of surface tension on a steep gravity wave. He argued that while the effects of surface tension may be small in an overall sense when compared with the effect of gravity for waves that are sufficiently long (hence the name gravity wave), they can still be locally important near the crest of a steep gravity wave because of the large curvature at that point. Based on this concept, he performed a perturbation expansion in which surface tension was taken to be the small parameter, and the steady, nonlinear gravity wave solution near the Stokes limiting wave form was used as the zeroth-order solution

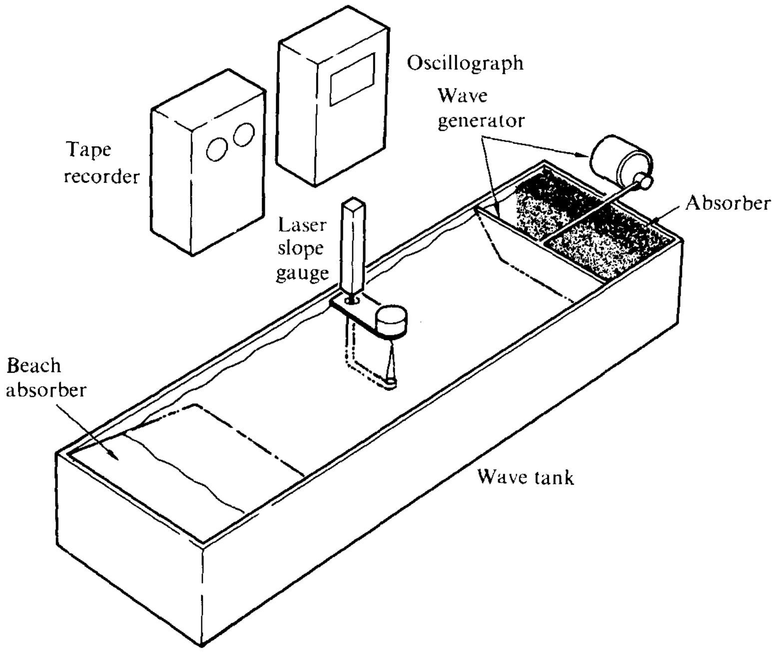

FIGURE 1. Experimental configuration.

to which the surface tension induced corrections were calculated. He found that the theoretical results compared favourably with the experimental observations of Cox in a qualitative sense. Because of the irregularities in Cox's data, the quantitative validity of the theory was not conclusively demonstrated.

Several more theoretical investigations into the interaction between gravity waves and capillary waves followed the analysis of Longuet-Higgins (1963), notably those by Crapper (1970), McGoldrick (1972), Benney (1976) and Ferguson, Saffman & Yuen (1978). Results from these studies were not in total agreement, presumably because of the fact that each analysis had treated a different set of physical conditions (see the discussion in Ferguson *et al.* 1978).

In this paper, we present a set of carefully controlled experiments to provide quantitative information on the behaviour of gravity waves (which are sufficiently long) for a range of values of wave slopes in the absence of wind. The high frequency capillary waves present on the gravity waves are resolved by a laser optical slope gauge. In §2, a brief description of the experimental facilities and the laser slope gauge is given. In §3, the results of the measurements are presented and discussed. In §4, comparison is made with the theory of Longuet-Higgins (1963) which is most appropriate for our experimental conditions. Finally, a summary of our findings is presented in §5.

2. Experimental apparatus

The experimental apparatus consists of a water tank in which deep-water gravity waves are steadily generated, a laser slope gauge which measures the slope of the gravity and capillary waves, and a tape recorder and oscillograph system which records the data. A sketch of the system is shown in figure 1.

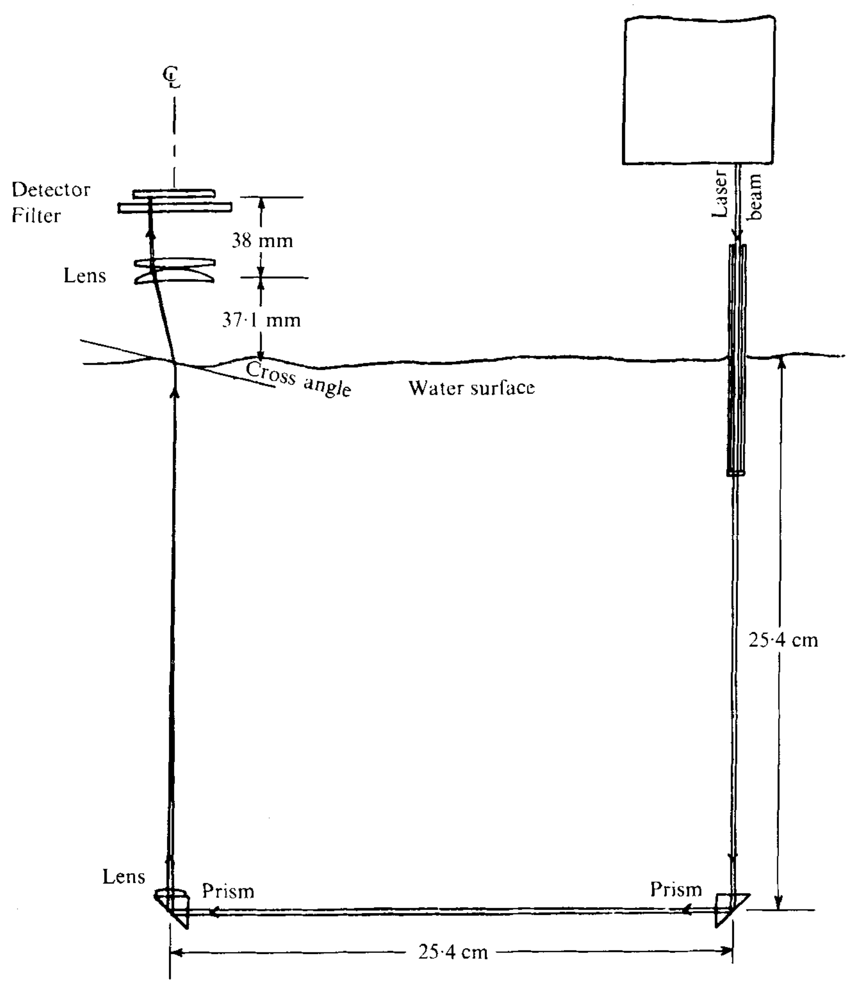

FIGURE 2. Optical schematics of laser slope gauge.

The water tank is 366 cm long, 30·5 cm deep and 61 cm wide. Surface gravity waves of varied and controlled frequency (range = 3 Hz–6 Hz) and amplitude (range = 0 cm–3 cm) are generated by a flapper plate driven by a variable speed motor. These waves propagate down the tank until they are absorbed by the inclined metallic plate (estimated reflexion less than 5 %).

The prime diagnostic is a laser slope gauge which is capable of measuring the slope changes of the high frequency capillary waves without probe interference. The principle of operation for this device is straightforward. A laser beam is inserted into the water, reflected upward normal to the surface which refracts it in a direction dependent upon the slope of the water surface. The refracted beam is directed by a lens to a position on the detector dependent only on the refracted beam angle. The magnitudes of the electrical outputs from the detector are dependent upon the incident light spot position and provide a measure of the water slope.

An optical schematic of the gauge is shown in figure 2. All of the optical components are mounted in a single rigid structure to maintain optical alignment. The light source is a helium–neon laser. The laser beam enters the water through a window at the bottom of a tube located 25 cm away from the point of measurement. The tube eliminates the effect of water surface orientation on the beam entrance angle. The beam is deflected horizontally and then up by two prisms. A small lens is cemented to the face of the second prism to focus the laser at the water surface. As the beam leaves the water it is refracted. An objective lens with a 34·5 mm aperture then focuses the beam on the detector. The detector is located in the focal plane of the objective lens so that the position where the laser beam intersects the detector surface depends only on the refraction angle and is independent of where the beam enters the lens, hence insensitive to water height. The narrow band filter centred at 6328 Å is used to filter out ambient room light. The detector is continuous and sensitive in two dimensions. The calibrated electric output provides a quantitative measure of the wave slope in two dimensions. An integrator circuit provides a measure of the surface wave amplitude. Experiments have demonstrated that this laser slope gauge is responsive to frequencies up to 700 Hz. It has a spatial resolution of 0·2 mm and a sensitivity of 0·01° change in slope.

3. Results and discussion

Returns of the laser slope gauge measurements were recorded on a magnetic tape and played onto an oscillograph in the form of time traces. An example of such an oscillograph output is given in figure 3. Time increases from left to right, so that the front of the wave is to the left-hand side. The lower trace in the figure marked 'amplitude' is the time integrated result of the slope gauge output, and it has been verified by comparison with conventional capacitance wave gauge records that it indeed represents a true record of the wave amplitude trace in time.

The point at which the slope trace crosses the line marked 0° gives the location of the crest of the wave, at which point the slope is zero by definition. The skewness of the slope gauge output reflects the fact that the nonlinear gravity waves are more peaked at the crest than at the trough. The maximum excursion of the slope output in both the positive and negative gives the maximum front and rear slopes, denoted by S_+ and S_-, respectively.

The steepness of a gravity wave is often characterized by the value it has for ka, where k is the wavenumber and a is the amplitude of the wave. In the present situation, we do not have a direct measurement of the wavenumber or the wavelength, but only the frequency of the wave. In the linear case, we know that the frequency and wavenumber are related by the dispersion law

$$\omega^2 = gk. \tag{3.1}$$

We acknowledge that for our case most of the waves are nonlinear, and therefore the dispersion relation (3.1) may not be valid. However, there is ample evidence that whereas the effects of the nonlinearity may be dynamically important, the correction to the numerical value of ω as a function of k is small (see Kinsman 1965). For this reason, we shall adopt the relation (3.1) in converting our frequency and amplitude measurements to a value of ka, using the formula (in c.g.s. units):

$$ka = \frac{(2\pi)^2}{g}\left(\frac{a}{T^2}\right) = 0{\cdot}0402\,\frac{a}{T^2}, \tag{3.2}$$

where T is the period in time of the wave.

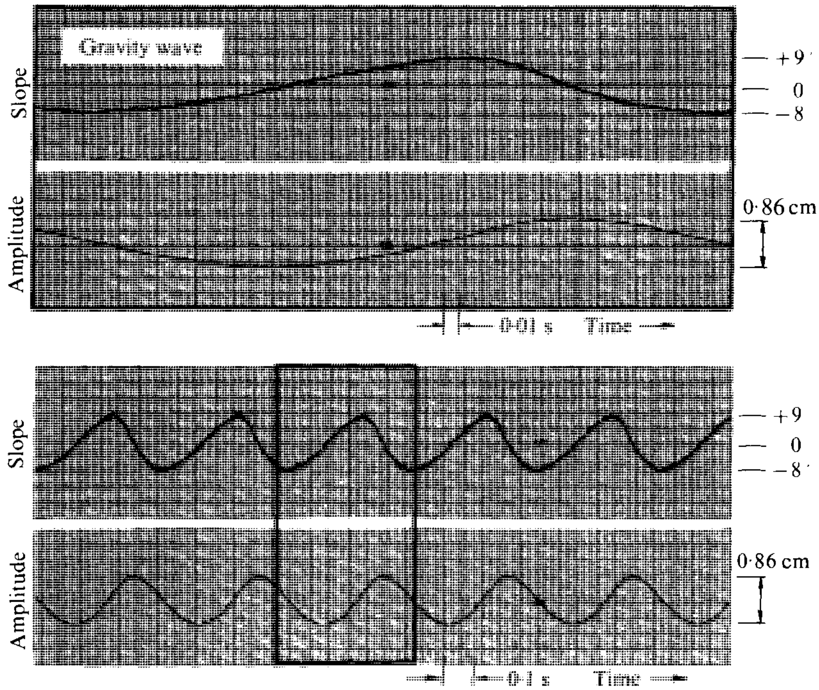

FIGURE 3. Oscillograph output for a small but finite amplitude gravity wave train.

A second example of the output is given in figure 4, in which high frequency capillary waves are seen near the crest at the front side of the gravity wave. This is typical for cases in which the gravity waves are sufficiently steep; in this case the value for ka is 0·236 using the definition in (3.2), with $S_+ = 0{\cdot}485$ and $S_- = 0{\cdot}268$. (Note that for the steady, progressive, symmetric Stokes limiting wave form, $S_+ = S_- = \tan 30^\circ = 0{\cdot}577$.) The fact that S_+ is much larger than S_- in this case indicates that the waves shown are highly asymmetrical, being much steeper in the front than in the rear. Since there has been no steady, asymmetric solution found, it is inferred that these waves are also unsteady and possibly undergoing wave breaking. This may account for the high degree of irregularity shown in the wave records. The quality of this set of records is similar to that published by Cox (1958) which, although positively identifying the presence of capillary waves on the front side of the gravity waves, is too irregular and unrepeatable to allow quantitative information to be deduced from it.

By making certain that the surface is clean and carefully selecting the wave frequencies to avoid possible cross-wave resonances, we were able to generate a much more repeatable set of gravity waves for fairly large values of ka. As we have indicated above, measurements of these waves were taken at a location 1·219 m from the wave paddle. It is hoped that this selection is far enough from the wave paddle to be relatively free of paddle induced flows, yet at a short enough fetch to ensure that modulational effects of the Benjamin & Feir type have not become significant (Benjamin & Feir 1967; Benjamin 1967; Lake *et al.* 1977). As a result, the waves can be made extremely repeatable, as shown by the example in figure 5. Of course, the mere fact that they are repeatable in time does not automatically guarantee that the waves are steady in space. Nevertheless, examination of the values of S_+ and S_- indicates that they are

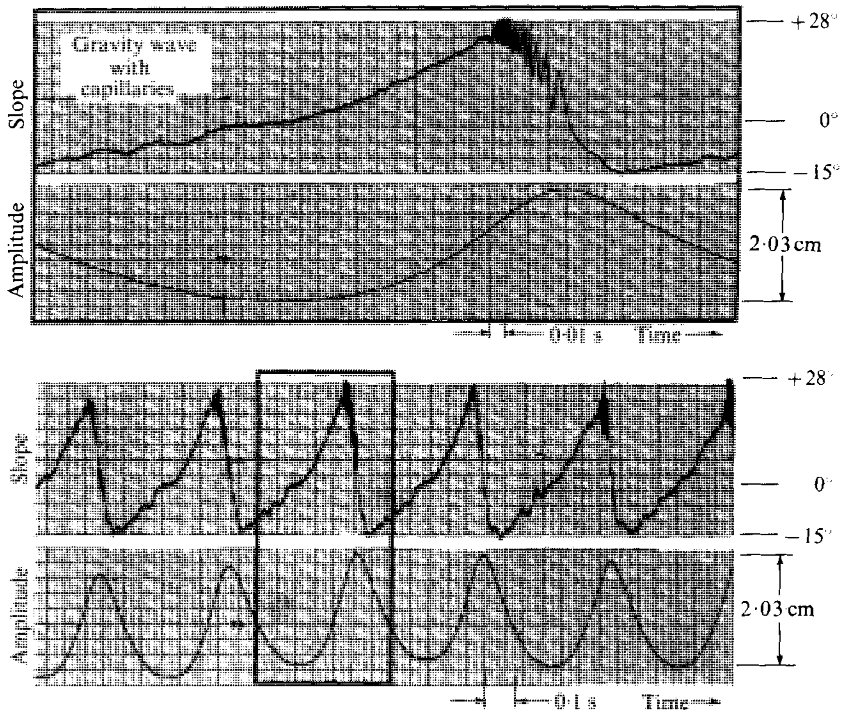

FIGURE 4. Oscillograph output for a steep gravity wave train generating high frequency capillary waves.

practically identical (provided that the value for S_+ is obtained by averaging out the high frequency oscillations caused by the presence of the capillary waves). This shows that the waves are symmetrical, and it is therefore reasonable to infer that they are steady. The ultimate determination of steadiness with our apparatus, of course, requires the use of two closely located slope gauges. This set of measurements is presently being pursued.†

All the data presented in the remainder of this paper are obtained from repeatable experiments such as those shown in figure 5. The values are taken from a single realization, but in view of the repeatability of the wave records, one would not be able to extract any more information by averaging or performing any other statistical techniques.

In order to resolve the high frequency capillary waves, we played back the signals recorded at high speed. An example is shown in figure 6. The frequency of the capillary waves is obtained from the period between two successive peaks in the slope trace. The magnitude of the capillary waves, as measured by their slopes, is slightly more difficult to obtain. Since the capillary waves are superposed on the gravity waves, the slope measurements do not have a true zero from which we can deduce S_+ and S_- as we did for the gravity waves. However, we are able to obtain a value for $S_m = \tan\theta$, where θ is defined as the total peak-to-peak excursion of the oscillation associated with capillary waves in degrees, as indicated in figure 6. In the limit of the small slope, S_m is

† Preliminary results indicate that for cases measured, the capillary waves and the gravity waves are indeed steady on the time scale of the capillary waves.

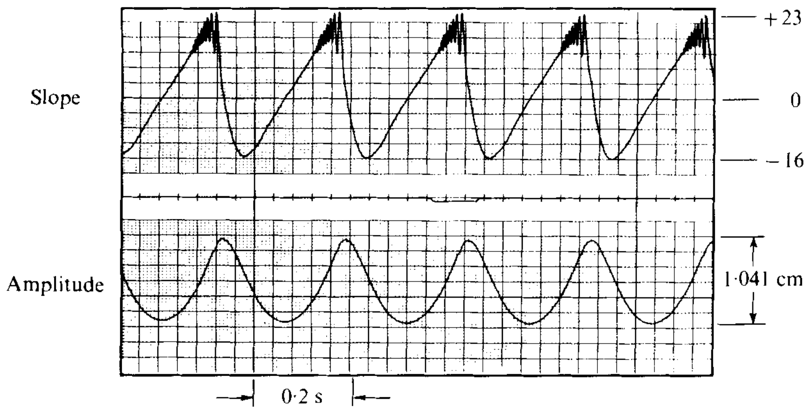

FIGURE 5. Oscillograph output for a repeatable train of gravity waves generating high frequency capillary waves.

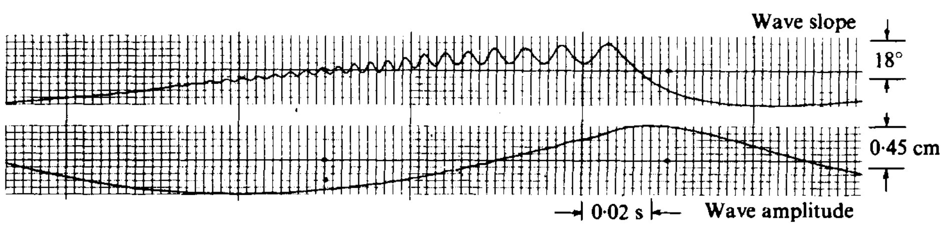

FIGURE 6. High speed tape recorder playback from which data were reduced. Gravity wave frequency, $f = 4{\cdot}21$ Hz.

expected to be the arithmetic mean of S_+ and S_-. However, for larger values of the slopes, S_m would be greater than the arithmetic mean because of the property of the tangent. It is true that S_m does contain contributions from the slope of the gravity waves, but it is deemed to be fair representation of the characteristic slope of the capillary waves to a good approximation.

It should be recalled that in the analysis of Longuet-Higgins (1963), the gravity wave was characterized not by its value of ka but by the value of its crest curvatures *in the absence of capillary waves*, κ_0. This idealized quantity cannot be obtained in an experiment, since surface tension (and hence capillary waves) is always present. The best approximation of such a value would be the crest curvature of the gravity wave in the presence of capillary waves. This is obtained by digitizing the slope record and numerically differentiating the slope record near the crest of the gravity wave (defined as the point of zero slope). Again, the dispersion relation must be invoked to convert a time differentiation to a spatial differentiation. The possible error introduced by the space–time interchange, however, should be small compared with the possible error associated with differentiation of the data, and the unknown contribution to curvature from the first capillary wave. To emphasize the fact that the resulting value is experimental, as opposed to the idealized crest curvature κ_0 (in the absence of surface tension) required in the Longuet-Higgins theory, we shall denote the former as $(\kappa_0)_{\text{exp}}$.

In the next section, we shall briefly summarize the essence of the Longuet-Higgins theory and compare its predictions with our experimental data.

4. Comparison with the theory of Longuet-Higgins

The effects of surface tension in the equations of motion for water waves is to modify the dynamic free-surface boundary conditions by introducing an additional surface pressure term:

$$\frac{\partial \phi}{\partial t}+\tfrac{1}{2}(\nabla\phi)^2+gz = p-T\kappa, \quad z=\eta(x,y,t), \tag{4.1}$$

where ϕ is the velocity potential, ∇ is the gradient operator, T is the surface tension coefficient, κ is the local curvature of the free surface, and p is the surface pressure. For our purpose, p can be taken to be a constant and hence zero by choice. The rest of the deep-water wave equations remain unchanged:

$$\nabla^2\phi = 0, \quad -\infty < z < \eta(x,y,t), \tag{4.2}$$

$$\frac{\partial \eta}{\partial t}+\nabla\phi.\nabla\eta-\frac{\partial \phi}{\partial z} = 0, \quad z=\eta(x,y,t), \tag{4.3}$$

$$\frac{\partial \phi}{\partial z}\rightarrow 0 \quad \text{as} \quad z\rightarrow -\infty, \tag{4.4}$$

where (x,y) are horizontal co-ordinates and z is the vertical co-ordinate.

The curvature κ can be expressed in terms of the free surface $\eta(x,y,t)$ as

$$\kappa = \frac{\eta_{xx}(1+\eta_y)^2+\eta_{yy}(1+\eta_x)^2-2\eta_{xy}\eta_x\eta_y}{(1+\eta_x^2+\eta_y^2)^{\frac{3}{2}}}. \tag{4.5}$$

For one-dimensional infinitesimal waves, where $\eta = a\cos(kx-\omega t)$, κ is simply

$$\kappa = -k^2 a\cos(kx-\omega t), \tag{4.6}$$

where a is the amplitude and k is the magnitude of the wavenumber. This leads to the linearized dispersion relation

$$\omega = (gk+Tk^3)^{\frac{1}{2}}. \tag{4.7}$$

The ratio of the effects of surface tension to gravity can be expressed by the dimensionless quantity

$$\tilde{\sigma} = Tk^2/g; \tag{4.8}$$

$\tilde{\sigma} \gg 1$ corresponds to capillary waves, and $\tilde{\sigma} \ll 1$ corresponds to gravity waves. For water, T is taken to be 74 c.g.s. units. This means that $\tilde{\sigma} < 0{\cdot}01$ for waves longer than 18 cm, and $\tilde{\sigma} > 100$ for waves shorter than 0·18 cm; these values conveniently define the gravity and capillary wave regimes respectively. In between, both gravity and capillarity play a role; they become of roughly equal importance for waves with wavelengths about 1·7 cm. These waves are sometimes known as Wilton's ripples, since it was Wilton (1915) who first gave a series representation for the steady-state finite amplitude solutions.

Longuet-Higgins (1963) was concerned with waves that are much longer than 18 cm and are pure gravity waves according to the foregoing discussion. Surface tension is expected to be important only locally near the crest. Thus an expansion of Wilton's type which requires $\tilde{\sigma} \sim O(1)$ is not suitable. The basic idea in the analysis of Longuet-Higgins (1963) was as follows: when the gravity wave amplitude increases, the curvature at the crest increases rapidly; since the surface tension effect is proportional to the local curvature, it must become locally important at the crest before the gravity wave achieves its limiting profile which has an infinite crest curvature corresponding to the sharp corner of 120°. The proposed expansion was therefore:

$$\phi = \phi^{(0)} + \phi^{(1)}, \quad \eta = \eta^{(0)} + \eta^{(1)}, \tag{4.9}$$

where $\phi^{(1)}$ and $\eta^{(1)}$ are $O(\tilde{\sigma})$ compared with $\phi^{(0)}$ and $\eta^{(0)}$. No limitation is imposed on the steepness of the gravity waves. Thus $\phi^{(0)}$ and $\eta^{(0)}$ simply represent the pure gravity wave, and the region of particular interest is when the wave approaches the Stokes limiting profile.

Substituting (4.9) into the governing equations (4.1)–(4.4) gives, to leading order, the pure gravity wave equations for $\phi^{(0)}$ and $\eta^{(0)}$, which are identical to the set of equations (4.1)–(4.4) when T is set to zero. To next order, we obtain the equations for $\phi^{(1)}$ and $\eta^{(1)}$:

$$\nabla^2\phi^{(1)} = 0, \quad -\infty < z < \eta^{(0)}, \tag{4.10}$$

$$\left.\begin{aligned} &\frac{\partial\phi^{(1)}}{\partial t} + \nabla\phi^{(0)}.\nabla\phi^{(1)} + \tfrac{1}{2}(\nabla\phi^{(1)})^2 + gz + T\kappa^{(1)} = -T\kappa^{(0)}, && (4.11)\\ &\frac{\partial\eta^{(1)}}{\partial t} + \nabla\phi^{(0)}.\nabla\eta^{(1)} + \nabla\eta^{(0)}.\nabla\phi^{(1)} + \nabla\eta^{(1)}.\nabla\phi^{(1)} - \frac{\partial\phi^{(1)}}{\partial z} = 0, && (4.12)\end{aligned}\right\} z = \eta^{(0)},$$

$$\frac{\partial\phi^{(1)}}{\partial z} \to 0 \quad \text{as} \quad z \to \infty. \tag{4.13}$$

These equations describe forced capillary waves riding on a variable stream induced by the gravity wave, the forcing being due to the curvature of the gravity wave. Longuet-Higgins (1963) then introduced the following additional assumptions:

(*a*) The gravity wave is steady, so that $\phi^{(0)}$ and $\eta^{(0)}$ are functions of $x - Ct$. In fact, an expression given by Davies (1951) was used for the description of the near limiting gravity wave around the crest.

(*b*) The capillary waves are small enough that the equations can be linearized about $\phi^{(0)}$ and $\eta^{(0)}$.

(*c*) The action of viscosity is such that it can be neglected for the capillary wave generation process, but large enough that the capillary waves, once generated, are damped out in one gravity wave wavelength.

Under these provisos Longuet-Higgins was able to solve the resulting equations by conformal mapping and Fourier transformation to arrive at the following conclusions:

(i) Capillary waves are generated at the crest of a sufficiently steep gravity wave. The steepness of the first (or typical) capillary wave is an extremely sensitive function of κ_0, which is the crest curvature of the zeroth-order gravity wave solution:

$$(ka)_{\text{1st cap}} \doteq \tfrac{2}{3}\pi\, e^{-\tilde{\lambda}}, \tag{4.14}$$

where

$$\tilde{\lambda} = g/(6T\kappa_0^2). \tag{4.15}$$

(ii) The frequency of the capillary waves is determined by the condition that the phase velocity of the capillary wave C_{cap} must be equal to the sum of C_{grav} and the gravity wave induced tangential particle velocity, i.e. $C_{\text{cap}} = C_{\text{grav}} + u_{\text{grav}}$, where u_{grav} depends on the frequency ka and the gravity wave, and is a function of position along the gravity wave.

(iii) The capillary waves are found ahead of the gravity waves in the direction of wave propagation. This is because the capillary wave group velocity $C_{g(\mathrm{cap})}$ is larger than the phase velocity C_{cap} [in fact, $C_{g(\mathrm{cap})} = \frac{3}{2}C_{\mathrm{cap}}$]. As a consequence, the pattern of the capillary waves generated must propagate forward relative to the gravity wave crest, radiating energy towards the front.

(iv) Internal viscous dissipation is considered to lead to a damping of the capillary wave energy at a time exponential rate of νk_{cap}^2 where ν is the kinematic viscosity and k_{cap} is the capillary wavenumber. According to assumption (*c*), this must be adequate to effectively damp out the capillary waves before they can reach the next gravity wave crest.

Longuet-Higgins (1963) compared these predictions with one experimental record published by Cox (1958). It was found that by taking the measured crest curvature as $(\kappa_0)_{\mathrm{cap}}$, the steepness of the capillary waves is underpredicted by the theory. No mention was made of the capillary wave frequencies, presumably because the records do not provide sufficient resolution to determine them accurately. Furthermore, the experimental record published, in which 13 gravity waves are shown, exhibits very large and irregular scatter, and no reliable quantitative information can be deduced from it. In that sense, one can only conclude that the comparison reveals no obvious contradiction between theory and experiment.

We now compare these theoretical predictions with our experimental data. We first examine the prediction of the capillary wave frequencies as a function of position along the gravity wave. The particle velocity induced by gravity waves is calculated by using equation (4.4) in Longuet-Higgins' paper, which is a consequence of the analytical expression for approximating gravity wave shape near the crest for near limiting wave forms given by Davies (1951). The results are shown in figure 7, which shows the capillary wave frequencies as a function of n, defined as the index of number of capillary waves from the gravity wave crest. As expected, good qualitative agreement is found near the crest of the gravity wave. The use of this measure (originally introduced by Longuet-Higgins) makes the comparison sensitive to the frequency and wavelength of the calculated capillary waves, since an error in the calculation will be cumulative as n increases. In view of this, the overall quantitative agreement, as shown in figure 7, should be considered satisfactory for the two values of gravity wave frequencies: 3·55 Hz and 4·21 Hz. The poor agreement of the 5·26 Hz case is expected, since for this case the gravity wave is not much longer than the capillary waves, as required by the theory. The relatively low frequency of the capillary waves also causes them to dissipate slower. In fact, the capillary waves cover the entire length of the gravity wave in this case, violating the third assumption of the Longuet-Higgins theory.

In order to test the validity of the theoretical prediction concerning the magnitude of slopes of the capillary waves, we have to obtain a value for κ_0, which is the crest curvature of the gravity wave in the absence of capillary waves. As we have indicated in the previous section, κ_0 is not obtainable from experimental measurements because of the fact that surface tension and hence capillary waves are always present. A logical approximation to κ_0 is $(\kappa_0)_{\mathrm{exp}}$, which is the experimentally measured crest curvature. The use of such a value to calculate the predicted capillary wave slopes, however, yields $(ka)_{\mathrm{cap}}$ several orders of magnitude lower than the measured capillary wave slopes. For example, take the case where the gravity wave frequency is 3·55 Hz;

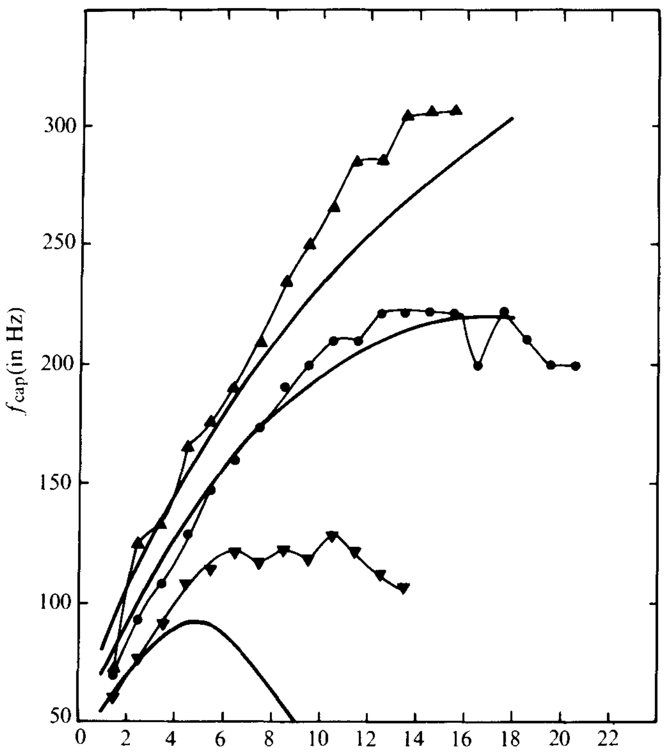

FIGURE 7. The distribution of high frequency capillary waves on a steep gravity wave. Capillary wave frequency f_{cap} *vs.* number of capillary wavelengths from gravity wave crest n for various values of gravity wave frequency f. ▲, 3·55 Hz; ●, 4·21 Hz; ▼, 5·26 Hz; ——, Longuet-Higgins (1963) theory.

the measured crest curvature is 0·356/cm, and use of this value in (4.14) shows $(ka)_{1st}$ to be $5{\cdot}6 \times 10^{-8}$, while the measured value of the first capillary wave slope is 0·07.

We note that because of the sensitivity of the dependence of $(ka)_{cap}$ on κ_0, a small reduction in the value of κ_0 may lead to an enormous difference in the calculated value of $(ka)_{1st}$. For this example, we note that in order to yield the value of $(ka)_{1st} = 0{\cdot}07$, the value required for κ_0 is 0·806/cm. In fact, the value of κ_0 required to achieve agreement between the calculated and measured value of $(ka)_{1st}$ is typically about twice that of the measured value of the crest curvature. It is therefore unlikely that this systematic discrepancy can be attributed entirely to the uncertainties in the measurement of crest curvature as we have mentioned above.

Actually, in the model proposed by Longuet-Higgins, the capillary waves are generated directly by the 'excess' pressure associated with the large crest curvature. It thus follows that the creation of these capillary waves drains the energy stored in the curvature of the gravity wave crest and reduces the value of the crest curvature. Since we are certain that our measurements are not taken at the instant of the first generation of capillary waves, there is reason to suspect that the measured crest curvature in the presence of the capillary waves should be less than the value which was 'responsible' for the presence of the capillary waves in the first place. It is therefore

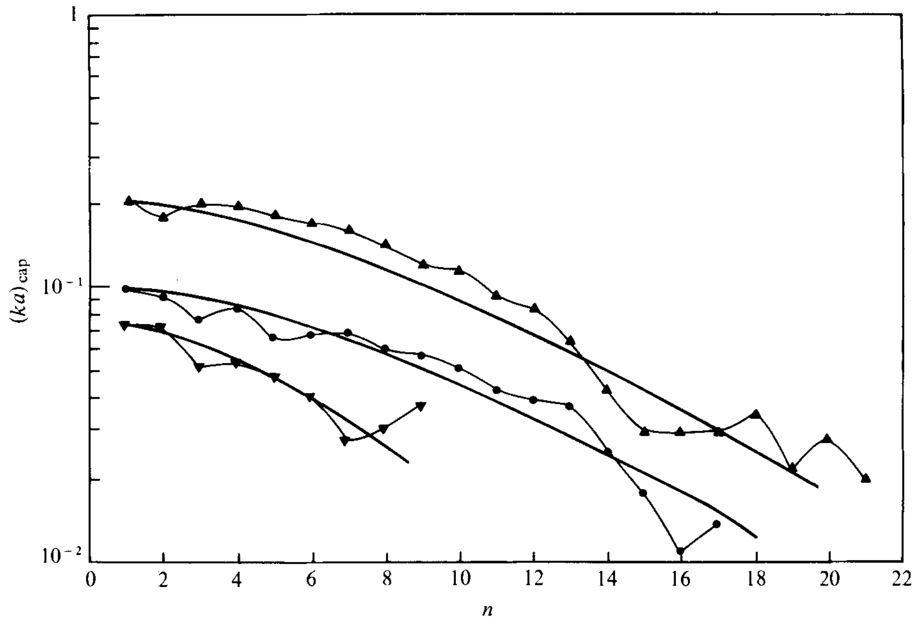

FIGURE 8. Dissipation of high frequency capillary waves on a steep gravity wave. Capillary wave slope $(ka)_{cap}$ *vs.* number of capillary wavelengths from gravity wave crest n for various values of gravity wave frequency f. ▲, 3·55 Hz; ●, 4·21 Hz; ▼, 5·26 Hz; ——, Longuet-Higgins (1963) dissipation model. The theoretical curves have been chosen to agree with the data for $n = 1$.

proposed that the present discrepancy, although substantial in terms of the magnitude of the capillary wave slopes, may be mainly caused by the slight mismatch of the theoretical assumptions and the actual experimental conditions. After all, the theory had been based on an ideal situation.

Finally, we can test the validity of the assumption in Longuet-Higgins' model that the effect of dissipation of the capillary waves is an exponential time-decay of wave amplitude given by νk_{cap}^2 where ν is the kinematic viscosity and k_{cap} is the capillary wavenumber. The results of this comparison are shown in figure 8. Since the magnitudes of the capillary wave slopes do not enter into the calculation, and we are interested only in the rate of dissipation, the origins of the theoretical curves are of no consequence. The curves in the figure are placed at a location most suitable for comparison with the experimental data. The agreement between theory and experiment indicates that it is a good assumption that capillary waves dissipate as free waves after their generation.

5. Conclusion

We have performed a set of repeatable experiments on the generation of high frequency capillary waves by steep gravity waves. The capillary waves were resolved with a laser optical slope gauge which has a frequency response of up to 700 Hz and a spatial resolution of less than 0·2 mm. Quantitative information on the capillary waves was obtained, and the data compared with the theory of Longuet-Higgins (1963).

It is found that most of the assumptions used by Longuet-Higgins are satisfied by the experimental conditions, and that the frequencies of the capillary waves and their dissipation rate once generated are well predicted. However, the relation between the capillary wave slope and the crest curvature of the underlying gravity wave predicted by the theory cannot be verified experimentally. The main cause of the discrepancy appears to be the fact that the theory is based on the knowledge of an idealized quantity, namely the crest curvature of the gravity wave in the absence of surface tension, a quantity which cannot be measured in an experiment.

A recent study by Ferguson *et al.* (1978) using a model equation to examine the effects of unsteadiness and viscosity on capillary wave generation by steep gravity wave suggests that viscous dissipation may affect the generation process in a more complicated way than that assumed by Longuet-Higgins. Whether or not their findings can be related to the discrepancy between theory and experiment observed in this paper must be determined by a more comprehensive theoretical and experimental study.

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